Theorem Examples

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This chapter demonstrates all the theorem-like environments available through the `callouty-theorem` and `custom-callout` extensions.

Theorems

Theorem 0.1 (Pythagorean Theorem)**.** *In a right triangle with legs of length a and b and hypotenuse of length c, we have:*

$$a^2 + b^2 = c^2$$

See Theorem 0.1 for the famous Pythagorean theorem.

Lemmas

Lemma 0.1 (Fundamental Lemma)**.** *Every non-empty subset of $\mathbb{N}$ has a smallest element.*

The result in Lemma 0.1 is used to prove many theorems in number theory.

Corollaries

Corollary 0.1 (Rational Root Test Corollary)**.** *If p/q is a rational root of the polynomial $a_n x^n + \cdots + a_1 x + a_0$ with integer coefficients, then p divides a_0 and q divides a_n.*

Corollary 0.1 follows directly from the Rational Root Theorem.

Propositions

Proposition 0.1 (Triangle Inequality)**.** *For any real numbers x and y:*

$$|x + y| \leq |x| + |y|$$

The triangle inequality (Proposition 0.1) is fundamental in analysis.

Conjectures

Conjecture 0.1 (Goldbach's Conjecture)**.** *Every even integer greater than 2 can be expressed as the sum of two prime numbers.*

Conjecture 0.1 remains one of the oldest unsolved problems in number theory.

Definitions

Definition 0.1 (Derivative)**.** The derivative of a function f at a point x is defined as:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided this limit exists.

See Definition 0.1 for the formal definition of a derivative.

Examples

Example 0.1 (Solving a Quadratic Equation)**.** Solve $x^2 - 5x + 6 = 0$.
We can factor this as $(x-2)(x-3) = 0$, so the solutions are $x = 2$ and $x = 3$.

Example 0.1 shows how to solve a simple quadratic equation.

Exercises

Exercise 0.1 (Integration Practice)**.** Compute the following integral:

$$\int_0^1 x^2\,dx$$

Try Exercise 0.1 to practice basic integration.

Proofs

Proof (Proof of the Pythagorean Theorem)

Proof of the Pythagorean Theorem. Consider a square with side length $a + b$. We can divide this square into four right triangles and a central square with side length c.
The area equations show:

$$a^2 + b^2 = c^2$$

□

Proof (Proof that the derivative of a constant is zero)

Proof that the derivative of a constant is zero. Let $f(x) = c$ for some constant c. Then $f'(x) = 0$. □

Solutions

Solution (Solution to Exercise on Integration)

Solution 0.1 (Solution to Exercise on Integration)*.* We use the power rule for integration:

$$\int_0^1 x^2\,dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$$

Solution (Alternative Solution to Quadratic Equation)

Solution (Alternative Solution to Quadratic Equation)*.* For the quadratic equation, we can use the quadratic formula:

$$x = \frac{5 \pm 1}{2}$$

This gives us $x = 3$ or $x = 2$.

Remarks

Remark (Convergence Note)

Remark 0.1 (Convergence Note)*.* The limit in Definition 0.1 may not exist for all functions at all points. For example, the absolute value function $|x|$ is not differentiable at $x = 0$.

Remark 0.1 highlights an important limitation of the derivative definition.

Mixed Example with Theorem and Solution

Here's an example that combines multiple theorem types:

Theorem 0.1 (Mean Value Theorem)**.** *If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists a point $c \in (a, b)$ such that:*

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Exercise 0.1 (Applying the Mean Value Theorem)**.** Let $f(x) = x^2$ on $[0, 2]$. Find the value of c guaranteed by the Mean Value Theorem (Theorem 0.1).

Solution (Solution to Mean Value Theorem Exercise)

Solution 0.1 (Solution to Mean Value Theorem Exercise)*.* We have $f(x) = x^2$, so $f'(x) = 2x$.
By the Mean Value Theorem:

$$f'(c) = \frac{f(2) - f(0)}{2 - 0} = \frac{4 - 0}{2} = 2$$

Since $f'(c) = 2c$, we need:

$$2c = 2$$

$$c = 1$$

Therefore, $c = 1$ is the value guaranteed by the Mean Value Theorem.

References